An interdisciplinary approach for mathematical education based on musical metaphors

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ABSTRACT

The Picalab Project proposes the design, development, and study of an integrated mathematics-music software solution to leverage learning of mathematics in a classroom context, by use of music as metaphors for mathematical curricular contents. Software modules were developed, based on Brousseau's Theory of Didactical Situations framework, and aimed at the 3rd, 4th and 5th grades of Chilean primary education level. We propose a three stage methodology to generate significant metaphors that link music and mathematics, generate functional prototypes as well as their respective teacher guide, and evaluation-feedback process to achieve a final version. Field and proof of concept tests were carried on at four public schools of different socio-economical profile in Santiago de Chile with 22 students (9-11 years old), for 60 minute long sessions. Preliminary results seem to indicate that this interdisciplinary approach is worthy of further research, which we expect to broaden as we gather more and definitive data in the quantitative and qualitative final assessment.

KEYWORDS: learning mathematics, music metaphor, Theory of Didactical Situations.

INTRODUCTION: MUSIC AND MATHEMATICS

Music has fascinated mathematicians and scientists, as a research subject, for centuries. Conversely, musicians have been attracted by the possibility of using mathematics in fields such as composition and analysis. Both parameterization of music in its physical-acoustical components and grouping/ordering musical units in sets have allowed musicians to create works based on evident mathematical planning [1]. The masterworks of Middle Age and Barroco's composers are a strong evidence of this. A similar argument can be provided in the case of the compositions based in twelve-tones combinations by Schöenberg, and for music based on parametric series (Boulez, Webern, Berg), on probability calculations (Xenakis) and randomization (Cage).

The relation between Music and Mathematics is not new but it has evolved through time. In classic Greece, music was not considered an art, but a science very close to mathematics. As it is known, it is not possible to speak about Pythagoras or Pythagorism, but only about a set of different authors' doctrines, the Pythagoreans, as they constituted a philosophical school and, moreover, a political and religious sect. For the Pythagoreans, music was not simply a human activity, but on the contrary, it maintained a central position on the Pythagorean metaphysics and cosmogony [2]. Some key concepts for these authors were: 1) the universe is a whole, ordered by a dynamic order because the cosmos is in movement. 2) The universe is also harmonic, that is, conciliation of opposites.

Harmony is a key concept for the Pythagoreans, and they understood it as a synthesis of opposites, like the concept of number, which is a synthesis of finites and infinites, even and odd…. In consequence, number and harmony as synthesis of contraries are immanent to everything and a basis for understanding. Music, both as a human activity and as a component of the cosmos, in the Pythagorean view, is composed by numbers. The deeper nature of harmony and numbers is unveiled through the music, e.g., through the proportions of sounding strings or the sound of tubes. This music-mathematics relationship has a near link with the power assigned to music: soul is harmony because its origin is the same. Music takes an ethical-pedagogical dimension: music is capable of producing catharsis. In this respect, music has the power to affect the soul because it is composed by numbers.

Therefore, music at the time was an abstract concept that doesn't strictly coincides with what we now understand as music. This line of thought was cultivated through Plato's and Aristotle's thought and prevailed until medieval times, and beyond. In the high middle age, music was conceived as a speculative art joining with the other three mathematical disciplines in the so called "Quadrivium": arithmetic, geometry, astronomy and music. Music had turned then into one of the "liberal arts", in both monastic and cathedral schools, and formed part of the educational curriculum of nobles and priests.

Perhaps the educational interest of mathematics relies in its capability to organize perception and thought. Maybe for this reason, mathematics has come to be a compulsory subject in general education, implemented as the cognitive structures of children change and evolve. This, in time, changes the perception of reality by means of the building of new mental representations, thus facilitating changes in the way of thinking.

The main reason we propose joining these two disciplines in our research is the synergy produced among both mathematical and acoustical elements of sound and music and the corresponding curricular school contents of mathematics. Integer and rational numbers, operations with numbers, graphical representations, randomness, combinatory and geometry can be treated by means of the interactions produced with mathematical elements of sounds and music (parameters and orders), acting as a powerful metaphor. We believe that music could facilitate the understanding of mathematical concepts due to the disciplinary proximity and the familiarity music has for students. But also, because sound and music is a strong motivator for a great number of students, in which case music can be used to leverage the acquisition of mathematical concepts. Furthermore, music can provide a significant, or ecologically valid context, to many of the abstract concepts from the mathematical discipline, where mainly the usual verbal, and at most kinesthetic and visual representations are used [3].

Some pedagogical advantages of using music and sound in learning mathematics are:

1. Music and sound act as an "apparel" element [4].
2. They anchor and contextualize new information in the social and cultural context of learning [5].
3. Potentially, they have the characteristics of a situated knowledge [6], [7].
4. Music and sound offer a new perspective of the mathematical phenomena, a new look. According to the cognitive flexibility theory, it is very important to give pupils opportunities to develop their own representations of the information in complex learning from different contexts or disciplines [8].

The Picalab (Program of Innovation in Art and Science) project's aim is to design, create and evaluate MMSI (Musi-Matemáticas Sonoras Interactivas, Sound Interactive Music-Mathematics), a set of didactic modules for mathematical learning in the Chilean Primary Education. MMSI includes software and printed materials that relate to the software (didactic guides). In the process of building MMSI, we have adopted Brousseau's Theory of Didactical Situation (TDS) framework, a cognitivist theory based that focuses on social and situated learning of mathematics [9]. Our contribution has been to relate mathematical concepts to the mathematical aspects of sound and music, with the purpose of giving better opportunities for learning mathematics.

The rest of the article is structured as follows: in the next section, we provide evidence of the strong effect that music has in the teaching of mathematics and revise some recent attempts of improving mathematical or science education through the use of music. Then we present our design methodology that leads to the creation of our MMSI modules.

In the following section we describe two of these modules, providing examples of activities and applications. Finally, we present the main conclusions of our work.

BACKGROUND

According to conventional wisdom, music and mathematics are related, and musical individuals are also mathematically inclined. After all, musical rhythm is based upon mathematical relations, and it is certainly reasonable to assume that an understanding of music requires some understanding of ratios and repeating patterns [10].

As Fiore [11] states, music and mathematics are indeed intricately related. Strings vibrate at certain frequencies. Sound waves can be described by mathematical equations. The cello has a particular shape in order to resonate with the strings in a mathematical fashion. After all, mathematics is the language that physicists utilize to describe the natural world and all of these things occur in the natural world. Not only do physicists, chemists, and engineers use math to describe the physical world, but also to predict the outcome of physical processes.

Music enhances spatial-temporal reasoning skills, which are crucial for learning concepts in proportional reasoning and geometry, areas in which students usually show below-average achievement [12]. Patterns are essential to both mathematics and music. Work with patterns enhances the thinking and reasoning skills of children because they must analyze a pattern to figure out its rule, communicate the rule in words, and then predict what comes next in the pattern. To translate a pattern, children keep the same rule but express it using a different medium. For example, a one-two-one-two pattern becomes a skip-hop-skip-hop pattern. Music patterns, such as the repeating melodies or refrains of a song or the beat of a rhythm, prepare children for a variety of number patterns, such as the sequence of odd and even numbers [13].

Therefore, if music is based on mathematical principles, and if an understanding of music requires some understanding of these principles, then it is possible that music education can lead to an improved understanding [10]. "Math and science tend to be stronger in students who have a music or an arts background" (Jensen, cited in [13]). Humans have created multiple sign systems to express and construct meaning. These sign systems increase our ability to express what we know in multiple ways. Language, music, art, and mathematics are all examples of these multiple communication systems. We can use the signs and symbols of the music and mathematics sign systems to help children explore this important symbol-human connection (Berghoff, cited in [13]).

Policymakers and industry leaders are calling for a 21st century education that is more interdisciplinary in nature, including the ability to solve problems and think creatively. Traditional teaching practices that present subjects as separate and distinct disciplines do not encourage students to make connections between subjects in school and in the inherently interdisciplinary nature of their daily lives [14].

Increasingly, teachers are being encouraged to engage in interdisciplinary instruction [13]. They argue that although many of us are comfortable using children's literature as the basis for interdisciplinary units, we rarely think to integrate mathematics and music in our lessons. Music actively involves students in learning and helps develop important academic skills (Rothenberg, cited in [13]). By using music to enhance children's enjoyment and understanding of mathematics concepts and skills, teachers can help children gain access to mathematics through new intelligences (Gardner, quoted in [13]). This integration is especially effective with children who have strong senses of hearing and musical intelligence.

Music instruction at the elementary school level includes helping students learn about melody, rhythm, timbre, and harmony, along with finding patterns and tones. Unfortunately, music is often taught in isolation from other disciplines. In a similar vein, content from physical and life sciences have traditionally been taught separately at the elementary school level, yet the natural connections between the physics of sound and the sounds in nature link these two areas of science and science with music [14].

A first step towards an educational integration between music and mathematics is to consider whether musical training is helpful for mathematical reasoning. There is evidence that musically trained students perform better in math. Vaughn [10] conducted a survey of studies providing direct evidence of the hypothesis that training in music results in improvements in mathematical performance. In the general discussion of this survey, three questions were asked: 1) Do individuals who voluntarily choose to study music show higher mathematical achievement than those who do not so choose? 2) Do individuals exposed to a music curriculum in school (not voluntarily selected) show higher mathematical achievement as a consequence of this music instruction? 3) And does background music heard while thinking about math problems serve to enhance mathematical ability at least during the music listening time? According to Vaughn, the answer to the first two questions is a clear and definitive yes, while the answer for the last questions, that is also a yes, is not so strong.

An, Ma and Capraro [15] conducted an exploratory research investigating the integration of music and a mathematics lesson as an intervention to promote pre-service teachers' attitude and confidence and to extend their beliefs toward teaching mathematics integrated with music. They randomly selected thirty students from 64 pre service teachers in a university. A 90-minute mathematics lesson integrated with a music composition activity was taught. Pre and post questionnaires were provided to evaluate the change in pre-service teachers' attitude and beliefs toward mathematics. The results demonstrated that the mathematics lesson integrated with music had a positive effect on pre-service teachers' attitude and beliefs toward mathematics teaching and learning.

Carrier et al. [14] conducted a study that examined the experiences of a teacher team: two elementary school teachers, a music teacher and a science teacher, as they developed and implemented innovative, interdisciplinary curriculum that combines physical and biological sciences of sound and animal communication with concepts from the discipline of music. This project involved designing curricula to provide opportunities for elementary school students to gain a deeper understanding of their world, expanding beyond the traditional classroom presentation of music and the physical properties of sound.

Courey et al. [16] examined the effects of an academic music intervention on conceptual understanding of music notation, fraction symbols, fraction size, and equivalency of third graders from a multicultural, mixed socio-economic public school setting. Students were assigned by class to their general education mathematics program or to receive academic music instruction two times/week, 45 min/session, for 6 weeks. Academic music students used their conceptual understanding of music and fraction concepts to inform their solutions to fraction computation problems. The results revealed statistically significant differences between experimental and comparison students' music and fraction concepts, and fraction computation at posttest with large effect sizes. Students who came to instruction with less fraction knowledge responded well to instruction and produced posttest scores similar to their higher achieving peers.

Johnson and Edelson [13] implemented activities for teaching children to express mathematical ideas, such as patterns and ratios, with physical materials, such as musical instruments. These activities range from use of musical symbols to illustrate serial order and fractions, data gathering for charts, use of sound to expand the concept of serial order, sorting and classifying, to determining ratios by means of measuring instruments of similar shape but different sizes, or fraction representation through duration of notes.

As a summary, Johnson and Edelson found that many good reasons exist for using music to help children learn mathematics. One reason is the broad range of significant concepts and skills that can be taught, such as recognizing, describing, and translating patterns; comparing and ordering the attributes of objects; representing data using pictures and graphs; and applying mathematics to everyday life. A second reason is the value that integrated mathematics and music activities have for children whose strengths lie in areas other than the logical-mathematical. A third reason is the ease with which even those of us who have a limited musical background can be successful with such activities. They finalize proposing that as teachers, we must take advantage of the many opportunities that music offers to help all our children learn mathematics in challenging and enjoyable ways [13].

An, Ma and Capraro [15] believe that by connecting arts or music into mathematics teaching and learning, pre service teachers may have more opportunities to change their beliefs about and attitudes toward mathematics. By designing appropriate music integrated into mathematics lessons, students can understand, analyze, and interpret mathematics through different routes. This strategy allows students to present and understand mathematics in alternative ways, especially for those who have a high level of musical-rhythmic intelligence.

DESIGN METHODOLOGY

The objective of the Picalab project is to design MMSI (Musical Mathematical Sound Interactive) modules, consisting of a software application paired with a didactic guide, which would allow a school teacher to present mathematical concepts or concepts, leveraged on a musical or sound based experience.

Great consideration was given to the fact that Math teachers do not necessarily have sufficient training in music, and could therefore be averted by the apprehension of having to address musical concepts they do not master during their lessons with the MMSI. To this end, a didactic guide was specifically written to show the teacher how to best take advantage of the interest that students naturally have in music and sound, to create a significant contextualization for otherwise abstract or difficult mathematical concepts.

In this sense, the MMSI were designed in a way that the teacher could be "allowed" to discover, along with the students, the sound and music aspects of the MMSIs, but on the other hand have a specific framework that would enable them to guide the classroom experience towards the specific mathematical concept addressed. Likewise, another great consideration is the large amount of curriculum that must be covered, with not too many weekly hours of math classes. Therefore, teachers cannot afford great amount of time, simply to enrich the context of any one specific content.

With all these considerations in mind, a framework was developed that allowed the production of a number of prototypes, which after a process of design, feedback and selection, would eventually converge to definitive MMSI modules that would comply to all the above pre-requisites.

The design methodology consisted of, in first place, producing as many musical-mathematical metaphors as possible. Recent findings in the field of cognitive neuroscience, point to the fact that greater chances of learning and comprehension of abstract mathematical concepts are achieved when provided with an ecologically valid context. Therefore, it should be a key feature in the design of any pedagogical activity, to always handle multiple representations, and of different nature (motor, kinesthetic, visual, hearing, linguistic, symbolic) [17]. These representations, or metaphors are the core of the MMSI, and were defined as any mathematical concept that could be mapped into a sound or music property. That is, a representation of a mathematical concept through a property or group of properties of sound or music. The key concept behind this methodology is that the comprehension of abstract mathematical concepts is greatly improved through the use of metaphors, that allows a student to represent the specific concept in a context that is significant to him/her. This new representation may be visual (as in a graph, were "higher" is correlated to greater cardinality), or kinesthetical (as widely used in the "number line", where displacement to the right represents a greater numerical value, whereas displacement to the left implies a lower value). In the case of MMSI, this same mathematical concept would be represented by sound: a higher pitch could represent greater value.

The production of MMSI consists of a three stage, iterative process. These stages are: Proposals for a non-functional prototype; Selection and prototype implementation; and Class evaluation and feedback.

For non-functional prototypes, a multidisciplinary team consisting of musicians, mathematicians, educators and scientists freely proposed metaphors that would embody a connection between mathematics and sound or music. These metaphors were produced freely, in an unrestricted manner, as long as the metaphor was clearly stated. Different possible activities were built around these metaphors. Since all members of the team should be able to add activities, notes, discussion and suggestions to each metaphor, an online collaborative document was setup for a period of 4 weeks, in which all team members could add ideas and comments to those presented, as well as continuously add new metaphors.

In order to organize and sort the collected material, a taxonomy was established to classify the potential non-functional prototypes. this taxonomy allowed the project team to classify every prototype under different criteria. The taxonomy defined was:

NAME: A tag that identify the proposal.
METAPHOR: specific metaphor used
AXIS: Curricular axis in which the proposal could be inserted.
AUTHOR: Name of the author of the proposal, to address the specific questions, clarify doubts, etc.
MATHEMATIC CONTENT: Mathematical topics addressed by the proposal.
MUSICAL CONTENT: Musical topics addressed by the proposal
LEVEL: School level in which the proposal could fit.

SELECTION AND PROTOTYPE DEVELOPMENT

The non-functional prototypes were organized, simplified and those that shared a common metaphor were merged into one single prototype. From that set only some prototypes were considered to be instantiated as a functional prototype. The criteria for the selection was, on one hand, the clarity of the metaphor, that is the a-priori evaluation of the impact of the metaphor in the acquisition of the desired mathematical content; and, on the other, the coherence of the proposed contents with the needs and curricula of the targeted school levels. Experts in the field subjectively evaluated both criteria.

MMSI: MUSIC-MATHEMATICS LEARNING MODULES

The proposed learning modules implement contents established by the Chilean National Curriculum and they are based on the didactic guidelines proposed by the Chilean Ministry of Education [18].

The MMSI modules are designed for the 3rd, 4th and 5th grades of the Chilean primary education, corresponding to ages 9, 10 and 11. Each module consists on a software component and a didactic guide. All of the MMSI modules were programmed in the Pure Data digital audio processing environment [19]. Now we present the basic components of some of the MMSI modules.

MultiPulse

MultiPulse (Multi-pulso) is a module that focuses on the recognition of multiple integer numbers on a game-like situation. In figure 1 it is possible to see the main interface and the configuration window. In this game, the positive integer series descends through the lines shown in the screen. The user must click on the correct keys when the descending number is an integer multiple of the number shown in color in the inferior row, being the possible options 2, 3, 4, 5 and 6. If the student makes the correct choice, a melody is heard in the previously selected musical scale. If the choice is incorrect, the system delivers non-harmonic sound as an error feedback. Like all MMSI modules, the activities are gradual and the protocol with which the teacher should work with this tool follows the TDS [9].

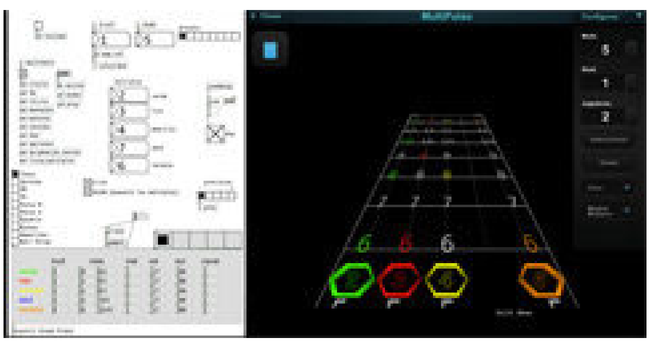

Figure 1: ***screenshots of the MMSI Multi-pulse module. (right) main interface is shown (left) configuration window in which can be set up several parameters (multiple number to play, number of simultaneous multiples to play, musical scale, accurateness, level, number of users, etc.)***

Students are encouraged to use the MMSI for the first time in an exploratory fashion. The role of the teacher here is crucial: it is he, as oriented in the didactic guide, who should compel his students to start elaborating conjectures regarding the behavior of the "game". This can be guided by certain key questions, which should make them start observing patterns, periodicity of keystrokes depending on the value of the base number, musical patterns that arise depending on the base number, etc. One illustrative example is that students can very quickly begin building models to predict when the keystrokes on two or more number "lanes" will coincide: in other words, students begin constructing a model to determine common multiple (as opposed to traditional lecture style, where the student has no participation in reaching this concept). In this way the concept of "least common multiple" is presented in a context that is significant to the student, and is the result of his or her own exploration, further enhanced by a musical experience.

Audiofractions

The Audiofractions (audio-fracciones) module, shown in figure 2, deals with rational numbers. In music, rhythm and pitch can be represented with simple fractions. In consequence, Audiofractions provides separated activities related to pitch and rhythm, which constitute its musical content. In terms of pitch, level 1 consists on exploration activities. According to the TDS, the beginning activities must be dedicated to exploration. This is the reason why a simple loop is presented to the students, where rational numbers must be determined using a vertical bar whose unit represents the fundamental sound. In level 2, the student is left to find the sound that is equivalent to a previously proposed one. Mathematically, the intention is to find a rational number equivalent to the given rational number, either based on multiplicative processes (sublevel A) or division processes (sublevel B). This is done with a bar with the representation and another bar that expressed the number or sound. In level 3, the students pursue a more creative activity by starting from a sound and fraction recorder, and then matching the numerical representations of each bar with a given sound pitch. The student creates and records his own composition as he makes selections of certain rational numbers that correspond to pitches in a musical scale. In order to develop the rhythmic aspects, the student can also use fractions in order to control the duration of the sounds, including also the possibility of producing silences.

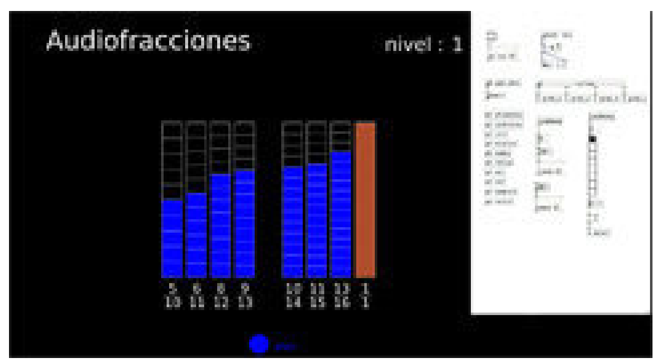

Figure 2. ***In this activity, students are requested to match a given ratio with equivalent ones (left, in red) adjusting the numerator and denominator on the two bars in the right. Each ratio corresponds to a sound, part of the harmonic series of a given sound, which is a note from a chosen scale in the configuration window (right, in white). Also in the first level of this activity, the ratio to match corresponds to a rhythm, which is a proportion of a music measure.***

As with the previous module, the teacher again plays a key role in using the musical incentive to promote the exploration of properties of fractions. In this case, a student can be given sound bars with different denominators (that is, bars divided into different amount of equal length parts), and the student is encouraged to equate the pitch of both. Once this is achieved, guided questions will quickly set the base to discuss, and experiment, with equivalent fractions. Likewise, a guided activity can help students compare fractions with different denominators, which they can corroborate by hearing the difference, helping them determine, for instance, which one is a greater fraction. The greatest achievement of the classroom experience, is when students begin formulating their own predictions, which they can formulate based on both the mathematical and musical aspects.

CLASS EVALUATION AND FEEDBACK

To correctly assess the impact that the use of MMSI in an actual classroom context produces, we conducted concept and usability tests of the modules in 4 public schools in Santiago, for 60 minute session, on 22 students (9-11 years old) from 4 public schools representative of the different socio-economical profile in Santiago de Chile. The 60 minute length allows for a full 45 minute classroom extension, plus extra time for feedback, reflection and interviews.

Figure 3. ***Pupils practicing with Multi-Pulse in the test sessions.***

Six MacBook-Pro computers were used for the sessions as shown in figure 3. Dialogs between students (sat down in pairs) and actions on screen were recorded while they were working with the corresponding module. Controllers were taking notes on each sessions at the four schools.

Pupils began working with the Multi-Pulse module in an exploratory way. In this first step, students freely explore the (musical) possibilities that the module has to offer. Their experience is mainly driven by achieving a musical outcome. It is important to note that, in this stage, the teacher has no need to impose an activity, and therefore has no need for any specific musical competence. This stage last for a few minutes, approximately 5 minutes or so. Pupils understood very fast (in the first minute) how the software worked, presenting no trouble at all using the computer keyboard to interact with MMSI.

Students are then guided to execute certain tasks on all the prototypes. That is, the teacher now sets a specific goal to achieve, or guides the activity by defining certain parameters and asking students to observe the results. Active discussion and formulation of hypothesis is encouraged, but usually surges spontaneously. A set of activities, covering the extension of the lesson, is provided as a "teacher guide", so as to avoid the lesson drifting away from its purpose.

Finally, we ask to pupils explain themselves the relationships between MMSI module and mathematics. In the specific case of Multipulse, as a result, students recognize regularity, relating multiplication to adding to the successive addition of pulses' duration. They applied this concept beyond multiples they know by heart (beyond 11 and 12's multiples), and they follow by counting rhythm pulses, so detecting the successive multiples. Pupils distinguish the concept of multiple, as the prominent numbers appear on the screen, which are the game point when click on them. When working with two simultaneous numbers, students distinguish the multiples of both numbers, distinguishing that common multiples are those that match in the two lines basis. The students come in a contextualized way to the concept of "minimum" common multiple, this being the lesser of the common.

Preliminary results show that students become highly motivated with this approach. Students show a very good attitude towards the modules, and remain in activities for the whole extent of the class. Most remarkable, is the fact that they can engage in active discussions about topics that, in a typical lecture format, they do not. They engage in formulating hypothesis regarding the "behavior" of different multiples, and then proceed to validate or reject them by means of the module itself. They consistently arrive at conclusions such as "a common multiple of two numbers is necessarily the product of these numbers", and shortly discover that this is not necessarily the least common multiple. The fact that these abstract or non-contextualized math topics are now presented in a musical context is apparently a key factor. This is currently being tested for later publication.

CONCLUSIONS

Summarizing, the analysis of the students' actions and data revealed a very important motivation and positive attitude towards the use of each music-mathematic module presented, particularly in those with a more game-like form. The music component, most evident in the exploratory (no guided) first phase of use of each module, is attractive to practically all students, even those that do not consider themselves "music experts". Equally important, this interest and motivation is also present towards the mathematical concepts involved. The fact that these are presented in a musical context seems to enhance interest and scaffold comprehension in them.

These preliminary results seem to indicate that this interdisciplinary approach is worthy of further research, which we expect to broaden as we gather more and definitive data in the quantitative and qualitative final assessment.

ACKNOWLEDGEMENTS

This research was funded by Fondo de Fomento al Desarrollo Científico y Tecnológico (FONDEF), Government of Chile.

The authors would like to especially thank all the members of the PICALAB team for their hard work on this project: Daniela Bravo, Freddy Chavez, Alexander Conde, Evelyn Herrera, Francisco Gonzalez, Gabriela Leigh, Pedro Lira, Paulina López, Roque Rivas, Federico Schumacher, Jaime Tello, Isabel Vargas, Alicia Venegas and Patricio Venegas.

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