Mathematics digital learning space: learning how to learn by cooperation

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ABSTRACT

The main aim of this article is presenting a cutting of the action-research which points out Facebook social network as a mathematics digital learning space that enables learning how to learn by cooperation, considering Jean Piaget's Theory reference. This research took place at IFRS – Campus Osório with 24 students from the 2nd High School integrated with Computing Technician course in 2012-1. Online digital technologies are present in the life of every student. Thus, they are attractive resources for schools in order to mobilize the learning process of each student – particularly mathematics, which finds an interdisciplinary context in these resources. The construction of concepts in mathematics is done by students' cooperative actions on Facebook, through the resolution of problems. While socializing diverse ways of thinking about the same math problem, students are allowed to make considerations about their own learning process, outdoing their own difficulties.

RESUMEN

El objetivo de este trabajo es mostrar un clip de la investigación-acción, señalando a la red social Facebook como un espacio de aprendizaje que permite a las matemáticas digitales de aprendizaje aprendizaje por cooperación, tomando como referencia la teoría de Jean Piaget. Esta investigación se llevó a cabo en la IFRS - Campus Osório con 24 estudiantes de la escuela de segunda en la computación técnica integrada en 2012-1. Las tecnologías digitales en línea están presentes en la vida de todos los estudiantes cada vez más por lo que estas características son atractivas a la escuela para movilizar el proceso de aprendizaje de cada estudiante, especialmente en matemáticas para un contexto interdisciplinario estos recursos. La construcción de los conceptos matemáticos que se haga de las actividades de cooperación por los estudiantes en la resolución de problemas a través de Facebook y socialización de diferentes formas de pensar el mismo problema de matemáticas permite a los estudiantes a reflexionar sobre su proceso de aprendizaje y superar las dificultades.

KEYWORDS

Cooperative Learning, Digital Technologies, Mathematics Education.

INTRODUCCIÓN

Everyday world is getting more dynamic and diverse, and it is difficult and challenging to the teacher, no matter in what knowledge area, to make his/her class attractive to the students, in basic school, technical school or college levels. Due to the imediatism verticalized in all educational environments, to Peters (2009) [5], this dynamism occurs because of the digital technologies around us everyday as a need not only of information, but also of communication. More specifically, Mathematics is a complex science to the students and it requires time so that the contents will be comprehended in a way students can look at them as something applied in their lives, no matter if it is in their daily and/or professional routine, according to D´ Ambrosio (1996) [9]. Students must demonstrate their mathematics comprehension making use of its language, its way to represent their thoughts and, therefore, build and interface of Mathematics application.

Integrating digital technologies and mathematics, a space of learning how to learn is created, and its main principle is the autonomy of the students in their own learning process. In example, through a simple learning object – applet – the student experiments and constructs a concept, little by little, according to the its pre-requisites, a new learning interface in which every time he/she interacts with him/herself and/or with collegues and the teacher, new strategies are established.

Observing, for about 12 years, the great difficulty of basic school students in mathematics, and, nowadays in Computing technical course, the "non comprehension of mathematics concepts and a simple repetition of meaningless procedures" idea was taken into account. In a master's research about Mathematics Portfolios, Bona (2010) [2] sustains that a learning how to learn mathematics space was found, making use of digital technologies as an interdisciplinary context for school's mathematics, according to the opinion of 290 students of a basic, public and state held school in Porto Alegre, in 2009.

From this point, the need of broadening and incrementing this study concerning a better comprehension of mathematics un learning process of each student, a mathematics digital learning space

is defined, mediated by the online digital technologies which potentiate cooperative learning, based on Jean Piget's theory – Reflexioning Abstraction (1977) and Sociological Studies (1973). This study adopts the action-research methodology from Barbier (2004) [1], developed in 2011 and 2012 with High School students from the integrated Computing technical course at IFRS – Campus Osório, in online or in person mathematics classes through cooperative actions in mathematics digital learning space. To Fiorentini and Lorezato (2007) [4], the research theme inserted in Computing in Mathematics Education is relevant concerning the importance of the use of digital technologies, but researches which point out to teaching practices of how to make use of this technologies in basic school are really needed.

Thus, this article is a cutting of this action-research held with 24 students from the environment described before, only the 2nd High School class, with the main goal of demonstrating the construction of mathematics concepts in a cooperative way among students in the digital learning space adopted: the social network Facebook. The main question of the research was: how to analyze and comprehend the process of cooperative learning process of mathematics concepts in the digital learning space?

The article is organized in: introduction, which also contains justification, theme, main question, objectives and methodology; followed by the definition of digital learning space and cooperative learning; then a spatial geometry problem solved in the context is elucidated; and after, the results, concluding remarks and references are presented.

MATHEMATICS DIGITAL LEARNING SPACE: FACEBOOK

The learning space used in schools nowadays is still the classroom only, the lab, the library and other particular environments of each teaching institution, where it is possible to find objects of physical work. However, the advance of online digital technologies leads us to think about a digital learning space, where objects of work are imaginary and unreal, for instance, mathematics geometrical solids built up in a free software named Poly. The solids can be constructed whether in paper or online, but the latter exploring allows a deitailed enriched visualization, faster than the real construction.

Papert (1994) [8] studies have already pointed out that when a student is interacting with the computer, he/she creates its micro-world, where curiosity can be explored according to his/her own learning rythim, making use of his/her previous acquired knowledge. Oh the other hand, Peters (2009) [5] remarks that a learning virtual space needs technological functions directed to pedagogical aspects, where from the interactions of computer technologies, multimedia and network new special technologies for communication, transmission, exhibition, search, access, analysis, storage, virtual reality and managing are raised up.

Thus, Bona, Fagundes e Basso (2011)[3] define digital learning space as a geographically non-located place where teaching-learning process occurs through organization and application of a pedagogical concept, based on communication, interaction, teacher's collaborative work with students and cooperative work of students among them and with the teacher.

This space needs to include the following characteristics: absence of limites through internet, absence of spatial disposition in many moments, opacity (creation of spatial concepts, such as simulation, associated to real space, and the possibility of relations among objects in this space), virtuality (digital representation of something real) and telepresence (non-physical presence of the teacher, students and other agents).

Digital learning space adopted by the 24 students at IFRS – Campus Osório in 2012/1 to learn how to learn mathematics was the Facebook social network; first, because it is an "online place" where everybody was subscribed, they knew how to use it and they could access it everywhere by the fact that it is in a network and it is free. Also, there was the option of creating closed discussion groups. Second, the way in which Facebook is programmed, by the use of comments, provided an interactive reading of students among them to solve mathematics problems, which enabled a more dynamic and cooperative learning among students and with the mathematics teacher. Besides, the applications provided by Facebook such as Docs allow the attachment of files in many formats like pdf., ppt. and other, and also the images and links enabled as posts; and the resources such as Events creation may be used as an agenda of school tasks; the collective group chat is saved so that all participants can access it as many times as they want, even if they did not take part in the chat, and also, if some student is not online, he/she can see everything the others did in other moments.

There are also other reasons still quoted by the students and by the teacher, but those pointed out before are enough to comprehend the proposal and Facebook also includes the former quoted characteristics and fits to the digital learning space definition of Bona, Fagundes e Basso (2011) [3].

It is also worth emphasizing that "digital" is used instead of "virtual" due to the comprehension of digital learning space as something that includes digital culture as part of the life of every student nowadays, and it is not restricted to the fact of being online or just mediated by online digital technology or not, because, in example, students can be doing a work in the classroom in groups and with the presence of the teacher using computers connected to the Internet, posting ideas in this space, therefore, this space is not only virtual – it is more than that: it is digital.

Mathematics digital learning space hereused is the social network Facebook, and it is different from the definition adopted for vitual community and general use social network, as in a virtual community, even with a common objective, someone may take part or not, and in the social network the bonds among people are looser than they are in a community, but in the space, when a closed group is created in the social network, an objective is being determined and an agreement among participants is being established. This agreement was raised from the students' needs of including terms in

the didactic contract already established by the mathematics teacher with the group for the classroom activities.

In this contract, the rights and duties of teacher and students during the classroom activities and those occurred in the digital learning space are described, for instance: "All students must take part in mathematics problems solving so that everybody will have the questions solved and understood in different ways"; "The teacher can take part in the problems solving inasmuch as there is a question for him/her, otherwise students must wait until all participants try to solve the problem together"; "Teacher must log into the space once a week, during 2 hours, in order to solve doubts through the chat if students need, or take part in the posts". This agreement/contract supports cooperative learning, as Piaget (1973) [6] states that cooperation among students does not occur without mutual respect and reciprocity among all the interactants. Thus, digital learning space mediated through online digital technologies provides a cooperative learn how to learn among students in the construction of mathematics's concepts.

COOPERATIVE LEARNING

Piaget (1973, p.105, 81) [6] states that "cooperate in action is operate in common, or it is, adjust through new operations (qualitative or metrical ones) of correspondence, reciprocity or complementarity, the operations executed by each of the partners", and "collaborate, however, is summed by the joint of actions realized isolated by the partners, even when they do it towards a common objective".

In the correspondence action, both students have their operations preserved, but in complementarity an addition of students' sequence actions occurs, and reciprocity requires the action of a student to be related to the action of other simetrical student, considering a common truth and different justifications – so that the common truth is the correspondence. Reciprocity is the most complex type of cooperation, as it requires that one student understands how the other classmate thinks, identifying both have different ways of thinking and different points of view, but both are correct.

Piaget (1973) [6] still highlights that cooperating constitutes a system of operations that allow themselves to adjust to each other, and these individual operations constitute a system of descentered actions that might be coordinated due to groups of operations of others, as if they were self produced. Human being is understood as a social being, thus, his intelligence is developed through social interactions, or it is, from his social actions. Considering Piaget points out intelligence socialization begins with language acquisition, before sensorimotor stage, this means that cooperation is present in all human developmental process.

The therm "adjust", used by Piaget, is essential in order to distinguish cooperation from collaboration. When students' actions are adjusted to each other, the starting point is what has already been done by one classmate, accepting or refusing the action of others.

This integration or denial occurs by elevated *réfléchissement*[s] [1] to a high level in each cooperative interaction.

Knowledge does not occur from practical action itself, but from what it can be abstracted from the action on the objects, from what can be comprehended from the objects and the actions. From this point on, action awareness, comprehension of the object's characteristics and meaning of what was experienced towards new operations or new knowledge level matters, and it includes abstraction.

Abstraction can be emphirical, when it is supported on physical objects or in material aspects of the own action, as movement, according to Piaget (1977) [7], and provides, in a certain way, a descriptive conceptualizion from the observation data found in the material characteristics of action. Reflected abstraction, on the other hand, for Piaget (1977) [7], is based on forms and all cognitive activities of the subject, such as action schemes, in order to take out certain characters and use them for other purposes, as new problems. Reflected abstraction is also the result of a reflective abstraction after it became conscient, which is usually supported in mathematics by demonstrations or generalizations – the most complex stage for students.

It is also important to emphasize that reflective abstraction has two essential aspects: *réfléchissement*, which is the projection of what was taken out of a standard inferred on a high level, and reflexion, which can be comprehended as reconstruction and reorganization of a mental act on the higher level of what was transferred from the lower one.

In each student's interaction in the mathematics digital learning space – Facebook, in this case – teacher can analyze and comprehend the mathematics learning process of each student, through the student's action. It is possible to find the kind of abstraction and the cooperation way developed by each student with the other classmates in the development of the mathematics problem.

From the students' learning process analysis, teacher can come up with strategies in order to eliminate students' difficulties, and this planning is possible because everything is registered indefinitely in the digital space. And if the teacher still has doubts about students' comprehension or not about a mathematics concept, a question can be asked in a particular chat conversation if that is the case, or in an opened conversation for the group, if the teacher notices a recurrent question from more than one student. It is important to highlight that learning how to learn translates the ability of reflection, analysis and awareness of what is known, being willing to change own concepts, searching for new information and acquiring new knowledges resulting from the fast evolution of science and technology and its influences on menkind development. This learning how to learn is what is aimed from the student to develop in basic school while doing math, which is increasingly applicable to his/her life.

[1] This term was created by Piaget [7], originally in French, and no correspondant word was found in English. The original word in French, *réfléchissement*, has usually been used.

SPATIAL GEOMETRY PROBLEM SOLVED COOPERATIVELY IN FACEBOOK

During this first semester of 2012, the contents worked with 2nd High School class that took part in this study were: trigonometry in the rectangle triangle and in the trigonometric circle, trigonometric functions, flat and spatial geometry (except pyramids). In all contents, several activites were proposed, such as: problems list, learning projects, exploring mathematics softwares, investigative researches in group and individually and others.

All the activities are usually done in groups of 3 to 5 components, except the tests and the final period portfolio – that are taken individually. The average grade of each period is 7 from 10. All activities proposed by the math teacher, by the classmates and by teachers of other disciplines are posted in the group of the class on Facebook, named I201. In order to elucidate the work, a spatial geometry problem will be described. This problem was developed in the group on Facebook during the evening time by 22 of the 24 students, with an average of 3 interactions of each student until the correct resolution of the problem was understood by everyone, and verified by the teacher as correct.

Students are identified by letters A, B, C, D, E, and so on even all the parents and tutors have signed the research's consentiment therm. From 22 students who interacted to solve the problem, some interactions of 5 students were selected to show up the comprehension of concepts like empirical and reflective abstraction, types of cooperation and also as an exemple of the study dynamics worked on Facebook.

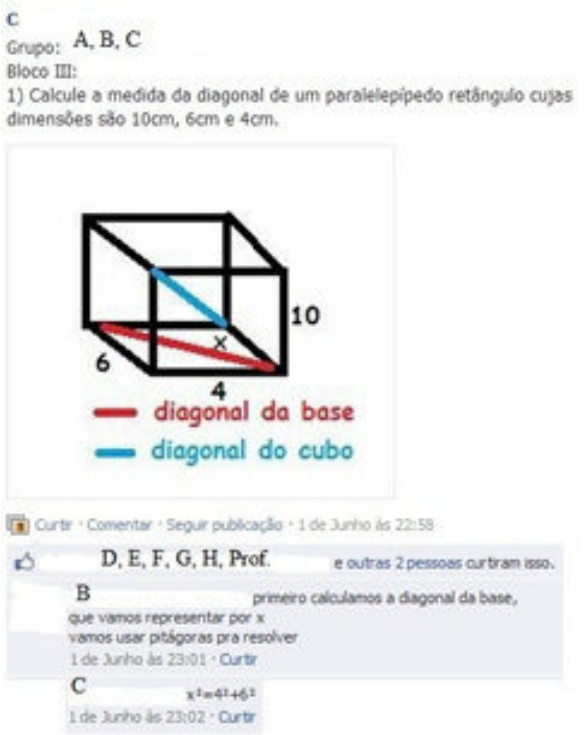

Figura 1.*Print screen of the problem posted on Facebook*

[2] Group: ABC
Block [of activities] III:

Continuing the interactions from Figure 1:

D: "olhando o desenho dá para ver o triângulo retângulo no chão para pitágoras, e que o risco vermelho é a hipotenusa, ne colega B[3]?"

It can be observed in D's interaction that he/she establishes an empirical abstraction from the representation constructed by C, and he/she asks B for cooperation by correspondence.

C: "É, D, isso...e vendo tb dá para ver outro pitágora com hipotenusa no azul que será a diagonal pedida, vc vê?"
B: "Sim, C, é isso daí as medidas são y^2 = 10² + x², onde y é diagonal pedida" [4]
The action of B in relation to C is cooperation by complementarity, in which the correspondence in agreeing occurs first, and second, more steps are added to the development of the problem solving.

D: "O x é raiz de 16 + 36 = 52. Não tira raiz pq ele para segundo pitágora, pode?"[5]

The action of D also cooperates with B by complementarity, but the following action of E with D is already a reciprocity action, because besides agreeing, it explains the result mathematically. It is also remarkable that symbolic math writing is adapted as Facebook does not provide mathematical symbols such as square root, and also the "Internet language" used online by students.

E: "Pode sim, D, pois a operação raiz quadrada e potencia de 2 são inversas, dai vale para poupar conta, só no fim tem de tirar raiz se der ou fatorar se não". [6]

The interaction of E shows his/her reflective abstraction in a non-elementary réfléchissement level, as the student establishes and explains the inverse operations and not only solve them mechanically.

1.Calculate the measure of the diagonal of a rectangle parallelepiped whose dimensions are 10cm, 6cm and 4cm.
red line: basis diagonal
blue line: cube diagonal
Student B: first, we calculate the basis diagonal, that we're going to represent by x
we're going to use Pythagoras to solve
Student C: $x^2 = 4^2 + 6^2$

[3] D: "looking at the picture, you can see the rectangle triangle on the floor for Pythagoras and the red line is the hypotenuse, isn't that right B?

[4] C: "That's it, D, that's it… and maybe also you can see another Pythagora with hypotenuse in the blue that's gonna be the asked diagonal, do u see?
B: "Yes, C, that's it and the measures are y^2 = 10² + x², where y is the diagonal asked"

[5] D: " The x is the root of 16 + 36 = 52. Don't make the root 'cause it stops according to Pythagora, huh?"

[6] E: "Yeah, u can, D, 'cause the operation square root and power of 2 are inverse, then it's worth to "save" calculation, just in the end u have to make the root if u can or make the factorization"

A: "Tb sei assim, E, e aproveitando (V52)², de D, + 100 = y² q dá y = raiz (152), q não tem raiz, né?"
C: " 152 = 2.2.2.19, raiz vai ser 2 sqrt 38 cm" [7]

Again, there is the action of cooperation by complementarity and reciprocity between A and C. The reflective abstraction demonstrated by D is also a réfléchissement applied to the context of measure units, because the student agrees with his/her classmates, but he/she wants to "visualize" the calculated value so that it will be meaningful in the representation of the parallelepiped with the other measures.

D: " É, C, raiz de 38 é aproximadamente 6,1 x 2 12,2 cm. Agora tem lógica, pois este valor dá dentro do paralelepípedo, com raiz eu não vejo o valor".
E: "Tava pensando que dá para fazer direto y = sqrt (10²+6²+4²) = 2sqrt 38 cm, é mais fácil e logico, não?" [8]

The process of reflective abstraction of student's E action, and also of student's C action is an indication of generalization, which is still confusing to student B, who questions student C as it follows:

B: "É, sim E, mas só vale se a variável for diagonal, se não é melhor fazer por partes, não?"
C: "Acho que não, B, vale sempre, é só elevar a diagonal ao quadrado e isola quem vc quer, entende?"
D: " Então dá para fazer geral y² = b² + H² + h², sendo y diagonal, b base, h altura plana, H altura espacial"
A: "Tudo em cm, D, ne?" [8]

Student D demonstrates the reflected abstraction principle, as he is conscient that generalization is validated in all cases in this way, and he cooperated with the other classmates in all the resolution. Student A cooperates with D by complementarity, because he/she adds the measure units.

Beyond these quoted students, other classmates "liked" the post and the interactions in the course of the resolution, meaning to say they agree and understand the same points mathematically speaking, because according to the students this is what "like" means on Facebook to mathematics.

All students understood how to calculate the diagonal of a parallelepiped and, consequently, of a cube as well, and all the problems were solved easily by the class students.

Concerning the two assessments in which these questions were, only one student was not able to solve them because he/she changed the value of the hypotenuse for a leg in the interpretation of the second Pythagoras – according to the logic of resolution given by these 5 students described formerly.

It is important to highlight that June 1st 2012 was a Friday night, and students go to school in the morning, so they were studying mathematics in the digital learning space by autonomy, every one cooperating among themselves and without the presence of the teacher, who checked the resolutions only on Monday – as this is the arranged day the class can access the teacher online.

The representation of the problem interpretation was done by the students using Paint software and posted on Facebook as a jpg. image.

There are many mathematics problems solved by the students on Facebook, demonstrating their autonomy and responsibility with their own mathematics learning process, and also a great interest in explaining the contents in many different ways to the classmates who had doubts. Besides, time destinated to mathematics regular classes in High School integrated with Computing technical course, due to the technical disciplines, makes the solving of any problem impossible to be done in class with the students. So, this digital learning space enables the students to study mathematics in real time and anywhere, and they still have access to the other classmates' way of thinking as another way to understand the content in a different perspective.

RESULTS AND CONCLUDING REMARKS

There are many results presented by the research and they are satisfactory, such as: students' construction of mathematics concepts; and the possibility of analyzing and comprehending how students construct these concepts mediated by digital technologies in cooperative actions among themselves, independent from the teacher. The definition of mathematics digital learning space is understood by students, parents/tutors and other mathematics teachers who have been adopting this innovative practice concerning the use of online digital technologies in basic education.

It was found that the idea of a mathematics digital learning space such as Facebook consists in an attractive element for students learning how to learn mathematics, together with the cooperative learning style. Students communicate and get information in the Internet with a dynamism and appropriation that would probably not mobilize each student in the same way, even if regular 2 period classes a week occurred, adding up to 1 hour and 40 minutes a week of regular classes and 2 hours in the digital space, in which the teacher is available for solving doubts and answering questions.

Students bring problems from different disciplines to be solved in the space, since they establish a relationship with mathematics, and these other actions demonstrate a certain appropriation of mathematics concepts, as knowing and showing the application is the first demonstration that the concept was comprehended.

[7] A: "I also know like this, E, and just for the record (V52)², from D, + 100 = y² in which y = root (152), that has not root, isn't it?"
C: " 152 = 2.2.2.19, root's gonna be 2 sqrt 38 cm"
[8] D: "Yeah, C, sqrt of 38 is approximately 6,1 x 2 12,2 cm. Now there's logic, 'cause this value is inside the parallelepiped, with sqrt I can't see the value".
E: "I was thinking that it's possible to go straight to y = sqrt (10²+6²+4²) = 2sqrt 38 cm, it's easier and more logical, no?"
[9] B: "Yeah, E, but it's only valid if the variable is diagonal, otherwise we'd rather do it in parts, no?"
C: "I don't think so, B, it's always worth, you just elevate the diagonal squared and isolate who u want, got it?"
D: "So u can do general y² = b² + H² + h², y is diagonal, b base, h flat height, H spatial height"
A: "Everything in cm, D, isn't it?"

REFERENCIAS

[1] Barbier, R. (2004). A Pesquisa-Ação, Liber Livro Editora, Brasília.

[2] Bona. A.S.D. (2012). Portfólio de Matemática: um instrumento de análise do processo de aprendizagem. Dissertação (mestrado). UFRGS - Pós-Graduação em Ensino de Matemática, Porto Alegre.

[3] Bona, A.S.D. & Fagundes, L.C. & Basso, M.V.A. (2011). A cooperação e/ou a colaboração no Espaço de Aprendizagem Digital da Matemática. RENOTE - Revista Novas Tecnologias na Educação, v.9, n.2, pp.1-10.

[4] Fiorentini, D & Lorenzato. S. (2007). Investigação em educação matemática: percussos teóricos e metodológicos. Auores Associados, Campinas.

[5] Peters, O. (2009). A educação à distância em transição. Unisinos, São Leopoldo.

[6] Piaget, J. (1973). Estudos Sociológicos. Forense, Rio de Janeiro.

[7] Piaget, J. (1977). Abstração Reflexionante: relações lógico-aritméticas e ordem das relações espaciais. Artmed, Porto Alegre.

[8] Papert, S. (1994). A máquina das crianças. Artmed, Porto Alegre.

[9] D'Ambrosio, U. (1996). Educação Matemática: da teoria à práxis. Papirus, Campinas.